HUMBOLDT-UNIVERSITÄT ZU BERLIN

BERNSTEIN CENTER FOR

COMPUTATIONAL NEUROSCIENCE

HUMBOLDT-UNIVERSITÄT ZU BERLIN
PHILIPPSTR. 13 HOUSE 6

PHONE: 030/2093-9110
FAX: 030/2093-6771
WEBPAGE: HTTP://WWW.BCCN-BERLIN.DE/

Models of Neural Systems, WS 2009/10
Project 1: Phase oscillators
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Background

Neural systems quite often reveal periodic dynamics e.g. regular firing neurons or oscillatory populations of neurons. The details of the dynamics of such models tend to be quite complicated which makes it hard to analyse them. Nevertheless, it can be shown that the dynamics in the limit of small perturbations can be reduced to so-called phase oscillators in which only the phase of the oscillation is modelled and the amplitude is assumed to be constant. The very simplest models has the following form:

$$\frac{\mathrm{d}\phi}{\mathrm{d}t}(t) = \omega \tag{1}$$

Here you will study synchronisation properties of such phase oscillators.

Problems

1. Implement a single oscillator perturbed by an external stimulus $I(t)$. Lets assume that the effect of the stimulus on the dynamics is phase-dependent, so that we can write:

$$\frac{\mathrm{d}\phi_i}{\mathrm{d}t} = \omega_i + I(t)\sin\phi_i \tag{2}$$

where the stimulus is a short pulse starting at time t_{onset} with duration T and intensity of I_0:

$$I(t) = \begin{cases} I_0, & \text{if } t_{\text{onset}} + T > t > t_{\text{onset}} \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

Plot phase-response curve i.e. the post-stimulus phase, as a function of the pre-stimulus phase for different I and t_{onset}.

2. Study the effect of the stimulus on a population of $N = 100$ identical phase oscillators (no coupling) with random initial conditions. Plot the phases as a color image. Calculate and plot the order parameter:

$$Z = \left| \sum_i^N \exp\left(j\phi_i\right) \right|, \tag{4}$$

where $j = \sqrt{-1}$ is an imaginary unit. What does the order parameter quantify?

3. Introduce white noise to the model:

$$\frac{\mathrm{d}\phi_i}{\mathrm{d}t} = \omega + I(t)\sin\phi_i + \sigma\eta(t), \tag{5}$$

where $\eta(t)$ is a normally distributed random variable of unit standard deviation and zero mean. Study the effects of the noise on the dynamics and synchronisation properties.

4. *Kuramoto mode* Simulate and analyse a population of globally coupled phase oscillators without noise. The dynamics of each oscillator is described by:

$$\frac{\mathrm{d}\phi_i}{\mathrm{d}t} = \omega_i + K/N\sum_{j=1}^{N}\sin(\phi_j - \phi_i), \tag{6}$$

where ω_i is an uniformly distributed random variable.

Under which conditions can the oscillators synchronize? Plot the order parameter Z as a function of the bifurcation parameter K.

5. It has been shown that oscillators can also form distinct clusters of synchronised activity. It can be achieved in a model when a non-trivial coupling is chosen. Read the paper Tass, Stochastic phase resetting of stimulus-locked responses of two coupled oscillators: transient response clustering, synchronization, and desynchronization. Chaos vol. 13 (1) pp. 364-376 and try to reproduce its results.

Richard Kempter Phone: 2093-8925 Email: r.kempter(at)biologie.hu-berlin.de
Robert Schmidt Phone: 2093-8926 Email: r.schmidt@biologie.hu-berlin.de
Bartosz Telenczuk Phone: 2093-8838 Email: b.telenczuk@biologie.hu-berlin.de