HUMBOLDT-UNIVERSITÄT ZU BERLIN
BERNSTEIN CENTRE FOR COMPUTATIONAL NEUROSCIENCE

Humboldt-Universität zu Berlin
Philippstr. 13 House 6

Phone: 030/2093-9110
Fax: 030/2093-6771
webpage: http://www.bccn-berlin.de/

Models of Neural Systems I, WS 2009/10
Computer Practical 6
Solutions to hand in on: November, 30th, 2009

Exercises

1. **Synaptic current**

   Simulate a linear membrane that receives an external synaptic input:

   $$\tau_m \frac{\mathrm{d}V(t)}{\mathrm{d}t} = -V(t) + E_m - r_m\, I_{syn}(t) \tag{1}$$

   where $I_{syn}(t) = g_{syn}(t)(V(t) - E_{syn})$ is the post-synaptic current, $g_{syn}(t)$ is the synaptic conductance. The change of the conductance due to a pre-synaptic spike can be described by:

   $$g_{syn}(t) = \begin{cases} g_{max}\, t/\tau_{syn} \exp\left(-t/\tau_{syn}\right) & \text{if } t \geq 0, \\ 0 & \text{if } t < 0. \end{cases} \tag{2}$$

   Here take τ_{syn}=10 ms, τ_m=10 ms, $r_m = 1\,\Omega\mathrm{m}^2$, $g_{max} = 0.5\,\mathrm{S/m}^2$, $E_m = -80$ mV.

   (a) Plot the following curves on one figure: synaptic conductance $g_{syn}(t)$, synaptic current $I_{syn}(t)$, membrane current $I_m(t) = (E_m - V(t))/r_m$, membrane potential $V(t)$. Consider inhibitory ($E_{syn} = -100\,\mathrm{mV}$) and excitatory ($E_{syn} = 0\,\mathrm{mV}$) synapses separately.

   (b) *Shunting inhibition*. In vivo neurons are constantly bombarded by balanced excitatory and inhibitory inputs. In order to model the background activity, assume that in addition to the time-dependent synaptic conductance (Equation 2) the membrane receives tonic inhibitory and excitatory inputs which can be best described by constant conductances. The total synaptic current equals:

   $$I_{syn} = g_{syn}(t)(V(t) - E_{syn}) + g_{exc}(V(t) - E_{exc}) + g_{inh}(V(t) - E_{inh})$$

   with $g_{exc} = 0.5\mathrm{S/m}^2$ and $g_{inh} = 2.0\mathrm{S/m}^2$. Show that the resting potential of the membrane doesn't change due to the tonic synaptic inputs. Simulate the membrane response to a transient excitatory synaptic input and compare it with the result from Exercise 1a. How would you explain the differences?

2. **Potassium channel**

The gate model was first introduced by Hodgkin and Huxley to describe voltage and time dependence of ion conductances in the squid axon. Today it is still the standard model of the ion current flow through transmembrane channels. One of its main assumptions is that the probability of opening and closing of an ion gate is described by the first-order kinetic equation:

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \alpha_n(V)(1-n) - \beta_n(V)n \tag{3}$$

where $\alpha_n(V)$ and $\beta_n(V)$ are voltage-dependent transition rates.

The potassium current in the Hodgkin-Huxley model is given by:

$$I_K = \bar{g}_K n^4 (E_K - V) \tag{4}$$

where $E_K = -77\,\mathrm{mV}$ is the reversal potential and $\bar{g}_K = 36\,\mathrm{mS/cm^2}$ is the maximum conductance. Rates $\alpha_n(V)$ and $\beta_n(V)$ are given by:

$$\alpha_n(V) = \frac{0.01(V+55)}{1-\exp(-0.1(V+55))}, \quad \beta_n(V) = 0.125\exp(-0.0125(V+65)), \tag{5}$$

where V is expressed in mV, and α_n and β_n are both expressed in units of ms^{-1}.

(a) Write a Python function defining $\alpha_n(V)$ and $\beta_n(V)$.

(b) Plot the steady-state activation $n_\infty(V) = \alpha_n(V)/(\alpha_n(V)+\beta_n(V))$ and the activation time constant $\tau_n(V) = 1/(\alpha_n(V)+\beta_n(V))$ in a voltage range of $-150\,\mathrm{mV} \le V \le 150\,\mathrm{mV}$.

(c) *Voltage clamp.* Calculate the current responses I_K to voltage steps under voltage-clamp conditions:

$$V(t) = \begin{cases} V_c & \text{if } t \ge 2 \text{ ms,} \\ -65\,\mathrm{mV} & \text{otherwise.} \end{cases}$$

with the initial condition $n(t=0) = 0.3177$. Plot I_K as a function of time. Repeat the simulation for different values of clamping voltage V_c from the range of -100 – -40 mV. What can be learnt from this experiment? What is the predicted effect of the potassium current on the membrane potential? Explain the obtained results referring to the plots of the activation variables n_∞ and τ_n.

(d) *Current-voltage relation.* Plot the I-V relation for the instantaneous and steady-state potassium current.

Contact

Richard Kempter	Phone: 2093-8925	Email: r.kempter(at)biologie.hu-berlin.de
Robert Schmidt	Phone: 2093-8926	Email: r.schmidt@biologie.hu-berlin.de
Bartosz Telenczuk	Phone: 2093-8838	Email: b.telenczuk@biologie.hu-berlin.de