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Models of Neural Systems I, WS 2009/10
Computer Practical 5
Solutions to hand in on: November, 23rd, 2009

Ordinary Differential Equations (ODEs)

Differential equations describe the evolution of systems in continous time and are widely used in science and engineering. Here, we will focus on the first order ordinary differential equations:

$$\dot{x}(t) = f\left(x(t), t\right), \tag{1}$$

where $\dot{x}(t) \equiv \frac{dx}{dt}$ is the first derivative of $x(t)$ with respect to time t. The solution to this equation is the function (dynamical variable) $x(t)$ which satisfies the above relation.

Exercises

1. **Numerical solutions to ODEs**

   The simplest numerical method of solving the equation (1) is by discretization. Lets assume that the function $x(t)$ is constant over short time interval Δt. We can then rewrite the equation using finite steps:

   $$\tilde{x}_{i+1} = \tilde{x}_i + f(\tilde{x}_i, i\Delta t)\Delta t, \quad \text{for } i = 0, 1, \ldots, N \tag{2}$$

   In order to find the solution, we start with the value $x_0 = x(0)$ given by initial condition and then proceed by adding small increments to the function according to the equation (2). This algorithm is called an Euler method.

   (a) Define a Python function which calculates and returns a numerical solution to any first-order ODE. The function should take as an argument the function describing the right-hand side of ODE $f(x, t)$, an initial condition x_0, an integration step Δt and a stop time $T_{\max} = N\Delta t$.

   (b) A population growth can be modelled with so-called logistic model:

   $$\frac{dx}{dt} = rx(1 - x) \tag{3}$$

   where r is a growth parameter.

   Using the Euler method solve the logistic equation with different initial conditions and values of the parameter r.

(c) Plot the numerical solutions to the logistic model with different integration steps Δt and compare them to the analytical solution:

(d) Calculate the mean square error between the numerical solutions $\tilde{x}_i$ and the analytical $x(t)$ solutions as a function of Δt:

$$\mathrm{MSE} = \frac{1}{N}\sum_{i=0}^{N}(\tilde{x}_i - x(i\Delta t))^2 \tag{4}$$

How does the accuracy change with the integration step Δt?

2. **Passive membrane**

The following equation describes a passive membrane, which is subject to a current injection:

$$\tau_m \frac{dV(t)}{dt} = -V(t) + E_m + R_m I(t) \tag{5}$$

At time $t = 0$ membrane voltage is at rest $V(t = 0) = E_m$.

(a) Build a model neuron from equation (5). Use $R_m = 10^7\,\Omega$, $I(t) = I_0 = 10^{-5}\,\mathrm{mA}$, $\tau_m = 10\,\mathrm{ms}$, $E = -80\,\mathrm{mV}$. Using Euler method find $V(t)$ as a function of time.

(b) Consider the following time-dependent current injection:

$$I(t) = \begin{cases} 10^{-5}\,\mathrm{mA} & \text{if } 100 \geq t \geq 10, \\ 0 & \text{otherwise.} \end{cases}$$

Find a numerical solution for $V(t)$. Repeat the calculations for several values of the membrane time constant τ_m.

3. **Integrate-and-fire neuron**

Linear models are unable to produce realistic action potentials, but spikes can be simulated in such models with a simple reset mechanism.

(a) Modify the Euler method so that every time when the membrane potential reaches the threshold $V_{\mathrm{th}} = -54\,\mathrm{mV}$ a spike is generated and the potential is set to the value $V_{\mathrm{reset}} = E_m$.

(b) Using the integration with reset simulate the neuron from Exercise 2 with a constant current intensity I_0. Show the voltage traces for 500 ms.

(c) Calculate and plot the mean firing rate as a function of input intensity. How do you interpret the results?

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