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Models of Neural Systems I, WS 2008/09
Project Assignment

Phase space analysis

Many features of dynamical systems can be understood by geometrical analysis of the underlaying equations. This method is called phase space analysis. Here we will considered a simplified two-dimensional system of spike generation proposed by Richard FitzHugh:

$$\dot{V} = V - V^3/3 - W + I \quad (1)$$
$$\dot{W} = 0.08(V + 0.7 - 0.8W) \quad (2)$$

Exercises

1. **Phase space**

   - Plot the vector flow field of the FitzHugh-Nagumo system on a grid V =-2.5 – 2.5, W =-1.25 – 1.25. Take $I = 0$. *Hint*: You can generate a 2D grid of points with `pylab.meshgrid` function; a field of arrows can be plotted in Pylab with `pylab.quiver` (check documentation for arguments).
   - Draw the W, V nullclines on the same plot. What does the vector field look like near the nullclines and their intersection?
   - *Stable points.* Starting at different initial conditions show that all trajectories go to the same point. Which point is it? Which initial conditions lead to the generation of an action potential? Can you find the potential (V) threshold above which spikes are reliably produced? *Hint*: You can read out the coordinates of your mouse cursor in the lower-right corner of the plot.
   - *Limit cycle.* Increase external current to $I = 1$ and plot several trajectories with chosen initial conditions. What has happened?
   - *Excitation block.* The FitzHugh-Nagumo model explains the excitation block phenomenon, i.e., the cessation of repetitive spiking as the amplitude

of the stimulus current increases. Plot sample trajectories for $I = 2$ and compare them with trajectories from the previous exercise. How can you explain the excitation block phenomenon?

- *Post-inhibitory spikes.* In Hodgkin-Huxley and Fitzhugh-Nagumo release from inhibition can also lead to a spike, called post-inhibitory spike. In order to simulate this phenomenon find (numerically or geometrically) the stable fixed point for inhibitory input (I=-0.5). Next, use the values as a (forced) initial condition in a model without external current (I=0) and show that it leads to generation of an action potential.

2. **Bifurcation diagram**

   In this exercise you will plot bifurcation diagram of FitzHugh-Nagumo model, which summarizes its solutions as the bifurcation parameter (external current, I) is varied.

   - *Fixed points* Find fixed points of Fitzugh-Nagumo model as a function of I by numerically solving the equation $V^3/3 - V + (V + 0.7)/0.8 - I = 0$. Take I =-0.5–2.5. Plot the value of the membrane potential (V) at the fixed point as a function of I. *Hint:* Use `scipy.optimize.newton(func, x0)` which finds zeros of a function defined by `func`. Take `x0 = 1`.
   - *Stability.* Determine stability of the fixed points numerically. Simulate the model with an initial condition very close to the fixed point (but not in the fixed point) and check if the trajectory stays in its proximity (stable point) or diverges (unstable point). Repeat the procedure for each fixed point. Plot the stable points with a solid line and unstable points with a dashed line.
   - *Stable limit cycles.* Determine the position of stable limit cycles. Start the integration from the initial condition $(V(0), W(0)) = (-10, 10)$. Plot the minimum and maximum of the trajectory as a function of input intensity I.
   - *Unstable limit cycles.* Determine the position of unstable limit cycles by repeating the last step but with integration time reversed ($t \to -t$). Use the initial condition $(V(0), W(0)) = (1., 1.)$.
   - Show all of the results on a bifurcation diagram.
   - Check what happens at the bifurcation point by narrowing the bifurcation parameter range (for example, $I = 0.32 - 0.34$). What type of bifurcation is it?

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