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Models of Neural Systems I, WS 2008/09
Project Assignment

Action potential propagation

The aim of the project is to model the propagation of an action potential along an axon. The relationship between the membrane current i_m and the voltage along an axon is given by the equation:

$$c_m \frac{\partial V}{\partial t} = \frac{1}{2ar_L} \frac{\partial}{\partial x} \left(a^2 \frac{\partial V}{\partial x} \right) - i_m + i_e, \tag{1}$$

where a is the radius of the axon, r_L is intracellular resistivity.

The ionic current flowing through a patch of axonal membrane i_m is well-described by Hodgkin-Huxley model:

$$i_m = g_{\mathrm{Na}} m^3 h (V - E_{\mathrm{Na}}) + g_{\mathrm{K}} n^4 (V - E_{\mathrm{K}}) + g_{\mathrm{L}} (V - E_{\mathrm{L}}), \tag{2}$$

where m, n, h are Hodgkin-Huxley-type activation variables.

Combining these two equations leads to a partial differential equation which can be computed numerically by multi-compartmental approximation. In a nonbranching cable, each compartment is coupled to two neighbours and the equations for the membrane potentials of the compartments are:

$$c_m \frac{dV_\mu}{dt} = i_m^\mu + \frac{I_e^\mu}{A_\mu} + g_{\mu,\mu+1}(V_{\mu+1} - V_\mu) + g_{\mu,\mu-1}(V_{\mu-1} - V_\mu), \tag{3}$$

where μ labels the compartments, I_e^μ is the total electrode current flowing into the compartment μ, and A_μ is its surface area. The constant $g_{\mu,\mu-1}$ determines the resistive coupling of the compartments and for nonbranching cables can be shown to be equal to $g_{\mu,\mu-1} = a/(2r_L L^2)$. This defines a system of ordinary differential equations which can be solved with Euler method and its modifications.

Problems

1. Numerically solve the cable equation for passive membrane (i.e. $i_m = (V -$

$V_{rest})/r_m)$. Compare the solution to the analytical solution. Take $r_m = 1\text{M}\Omega\text{mm}^2$, $r_L = 1\text{k}\Omega\text{mm}^2$.

2. Implement the Hodgkin-Huxley model of action potential propagation. Solve the partial differential equation using methods described in Chapter 6.6B of [1]. Take $r = 0.238$ mm and $r_L = 35.4$ Ωcm.

3. Initiate an action potential on one end of the axon by inserting a current in the terminal compartment.

4. Determine action potential propagation velocity as a function of the axon radius.

5. Generate action potentials from both ends of the axon. Show that they annihilate when they collide.

6. Simulate action potential propagation in a myelinated axon.

References

[1] P Dayan, LF Abbott, “Theoretical neuroscience”, MIT Press 2001

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