



Models of Neural Systems, WS 2008/09 Computer Practical 9

Hodgkin-Huxley model of action potential generation

Action potentials (aka spikes) are a dominant feature of nervous system and play crucial role in the computations performed by cortex, thalamus, cerebellum and associated structures. The ionic mechanisms of action potential generation were described on giant squid axon by Hodgkin and Huxley. The complete model they proposed is as following:

$$c_m \frac{dV}{dt} = -I_{Na} - I_K - I_L$$

with the ionic currents defined by:

- sodium current: $I_{Na} = \bar{g}_{Na} m^3 h (V - E_{Na})$
- potassium current: $I_K = \bar{g}_K n^4 (V - E_K)$
- leak current: $I_L = \bar{g}_L (V - E_L)$

The maximal conductances and reversal potentials used in the model are $\bar{g}_L = 0.3 \text{ mS/cm}^2$, $\bar{g}_K = 36 \text{ mS/cm}^2$, $\bar{g}_{Na} = 120 \text{ mS/cm}^2$, $E_L = -54.387 \text{ mV}$, $E_K = -77 \text{ mV}$ and $E_{Na} = 50 \text{ mV}$. The gating variables n , m , h follow first-order kinetics with the following rate functions:

$$\begin{aligned} \alpha_n &= 0.01 \frac{V + 55}{1 - \exp(-0.1(V + 55))} & \beta_n &= 0.125 \exp(-0.0125(V + 65)) \\ \alpha_m &= 0.1 \frac{V + 40}{1 - \exp(-0.1(V + 40))} & \beta_m &= 4 \exp(-0.0556(V + 65)) \\ \alpha_h &= 0.07 \exp(-0.05(V + 65)) & \beta_h &= \frac{1}{1 + \exp(-0.1(V + 35))} \end{aligned}$$

Exercises (adapted from [2])

1. Plot the steady-state activation and time constant of n , m and h as a function of voltage.

2. Simulate a Hodgkin-Huxley model by numerically integrating equations for V , n , m , h . Take $c_m = 1 \mu\text{F}/\text{cm}^2$ and as initial values take: $V = -65 \text{ mV}$, $m = 0.0529$, $h = 0.5961$, and $n = 0.3177$. Use Runge-Kutta method with a small integration step (test several values).
3. Use an external current with density $i_e = 20 \mu\text{A}/\text{cm}^2$ and plot V , I_K , I_{Na} as functions of time. Which currents are responsible for different phases of action potential generation?
4. Plot the firing rate of the model as a function of i_e over the range from 0 to $200 \mu\text{A}/\text{cm}^2$. Show that the firing rate jumps discontinuously from zero to a finite value when the current passes through the minimum value to produce sustained firing. (*Hint:* to detect spikes test when the membrane potential crosses a predefined threshold).
5. Apply a pulse of negative current with $i_e = -5 \mu\text{A}/\text{cm}^2$ for 5 ms followed by $i_e = 0$ and show what happens.

References

- [1] Ch Koch, "Biophysics of Computation", Oxford University Press, 1999
- [2] P Dayan, LF Abbott, "Theoretical neuroscience", MIT Press 2001, exercises available online at <http://neurotheory.columbia.edu/~larry/book/exercises.html>
- [3] D Johnston, S MS Wu, "Foundations of Cellular Neurophysiology", MIT Press, 1994

CONTACT

ROBERT SCHMIDT (ITB, R. 2316) PHONE: 2093-8926 EMAIL: R.SCHMIDT@BIOLOGIE.HU-BERLIN.DE
BARTOSZ TELENCZUK (ITB, R. 1309) PHONE: 2093-8838 EMAIL: B.TELENCZUK@BIOLOGIE.HU-BERLIN.DE