



## Models of Neural Systems I, WS 2008/09 Computer Practical 6

### Stochastic Models of Neural Activity

The responses of single neurons to external stimulations vary from trial to trial even if the same stimulus is presented. Therefore, it make often sense to model the neuronal activity by stochastic processes, such as Poisson process. In this exercise we will generate and analyse random spike trains generated from various distributions.

### Exercises

#### 1. Poisson Process

- (a) Generate a sequence of spike counts from Poisson distribution:

$$p(n) = \frac{(\lambda T)^n}{n!} \exp(-\lambda T), \quad (1)$$

where  $\lambda$  is a firing rate and  $T$  counting window width.

You can obtain spike counts from the Poisson distribution by transforming random numbers through the inverted cumulative Poisson distribution.

Take  $\lambda = 10$  Hz and several different choices of  $T$ . Plot the histogram of the obtained spike counts and show that it approximates well the Poisson distribution.

- (b) Simulate Poisson spike trains using Bernoulli approximation:

$$p(\text{spike in } [t, t + dt]) = \lambda dt. \quad (2)$$

Choose  $dt$  very small so that the  $\lambda dt \approx 0$ . Plot the resulting spike trains. Calculate spike count distribution in windows of different widths and compare the results to the histograms from exercise (a). Calculate Fano factor:  $F = \sigma_n^2 / \mu_n$  where  $\sigma_n^2$  is the variance of spike counts and  $\mu_n$  their mean.

- (c) Calculate interspike interval distribution of spike trains obtained in (b) and show that it follows exponential distribution  $p(\tau) = \lambda \exp(-\lambda\tau)$ . Calculate the coefficient of variation  $C_V = \sigma_\tau / \mu_\tau$ , where  $\sigma_\tau$  is the standard deviation of the interspike intervals and  $\mu_\tau$  their mean.

- (d) (*Optional*) Generate inhomogeneous Poisson process with firing rate given by  $f(t) = 100(1 + \cos(2\pi t/300))$ . Estimate time-dependent firing rate by calculating the number of spikes in a sliding window.

Hint: You can use `numpy.convolve` function.

## 2. Introducing refractoriness

- (a) Generate Poisson process as in 1b. Introduce absolute refractory period by simply removing spikes with interspike intervals shorter than the refractory period  $\tau_{\text{ref}}$ . Plot interspike intervals distribution. Repeat the simulations for different values of  $\tau_{\text{ref}}$ . Each time calculate the coefficient of variation  $C_V$ . What happens when  $\tau_{\text{ref}}$  is increased?
- (b) (*Optional*) Simulate spike trains with relative refractory period using Bernoulli approximation where probability of emitting a spike is given by:

$$p(\text{spike in } [t, t + dt] | t_{\text{last}}) = \lambda r(t - t_{\text{last}}) dt, \quad (3)$$

where  $t_{\text{last}}$  is the time of the last spike and  $r(t)$  is a recovery function. You can model the recovery function by  $r(t) = 1/(1 + \exp(-(t - \tau_{\text{ref}})))$ . Plot interspike intervals distribution. How do you interpret  $r(t)$ ?

## 3. Orientation-selective neurons in primary visual cortex (V1)

The spiking of cortical cells was shown to be very irregular. Therefore it can be often described by one of the stochastic models discussed above. In this exercise, we will implement a simple stochastic model of an orientation-selective V1 neuron.

- (a) Many neurons in V1 cortex were shown to modulate their firing rate to presentations of bars of different orientations. The dependence is described usually by so-called orientation tuning curve which can be modelled by the following function:

$$f(s(t)) = \lambda_{\text{max}} \exp[\cos(s(t) - s_{\text{max}})], \quad (4)$$

where  $\lambda_{\text{max}}$  is the maximum firing rate,  $s_{\text{max}}$  is the optimum orientation of the neuron.

Implement a Poisson model without refractoriness whose firing rate is given by Equation 4.

- (b) Simulate the response of the neuron to a bar rotating with a constant angular velocity. Repeat the simulation several times with the same stimulus. Plot a raster of the spikes elicited in each trial. Plot the post-stimulus time histogram of the spikes by binning the time axis and summing the number of spikes obtained in each bin over all trials.
- (c) What are the limitations of the model? How could it be improved?

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