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Models of Neural Systems I, WS 2007/08
Project Assignment
To hand in on Feb 11th 2008

Action potential propagation

The aim of the project is to model the propagation of an action potential along an axon. The relationship between the membrane current i_m and the voltage along an axon is given by the equation:

$$c_m \frac{\partial V}{\partial t} = \frac{1}{2ar_L} \frac{\partial}{\partial x} \left(a^2 \frac{\partial V}{\partial x} \right) - i_m + i_e, \tag{1}$$

where a is the radius of the axon, r_L is intracellular resistivity.

The ionic current flowing through a patch of axonal membrane i_m is well-described by Hodgkin-Huxley model:

$$i_m = g_{\mathrm{Na}} m^3 h(V - E_{\mathrm{Na}}) + g_{\mathrm{K}} n^4 (V - E_{\mathrm{K}}) + g_{\mathrm{L}}(V - E_{\mathrm{L}}), \tag{2}$$

where m, n, h are Hodgkin-Huxley-type activation variables.

Combining these two equations leads to a partial differential equation which can be computed numerically by multi-compartmental approximation. In a nonbranching cable, each compartment is coupled to two neighbours and the equations for the membrane potentials of the compartments are:

$$c_m \frac{dV_\mu}{dt} = i_m^\mu + \frac{I_e^\mu}{A_\mu} + g_{\mu,\mu+1}(V_{\mu+1} - V_\mu) + g_{\mu,\mu-1}(V_{\mu-1} - V_\mu), \tag{3}$$

where μ labels the compartments, I_e^μ is the total electrode current flowing into the compartment μ, and A_μ is its surface area. The constant $g_{\mu,\mu-1}$ determines the resistive coupling of the compartments and for nonbranching cables can be shown to be equal to $g_{\mu,\mu-1} = a/(2r_L L^2)$. This defines a system of ordinary differential equations which can be solved with Euler method and its modifications.

Problems

1. Implement the Hodgkin-Huxley model of action potential propagation. Solve the system with Euler method. Consider improved algorithms for solving PDE (reverse Euler, Crank-Nicholson). Take $r = 0.238$ mm and $r_L = 35.4\ \Omega$cm.

2. Initiate an action potential on one end of the axon by inserting a current in the terminal compartment.

3. Determine action potential propagation velocity as a function of the axon radius.

4. Generate action potentials from both ends of the axon. Show that they annihilate when they collide.

5. Simulate action potential propagation in a myelinated axon.

Contact

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