Models of Neural Systems I, WS 2007/08
Computer Practical 5
Discussed on 19th Nov 2007

Gated ion channels

The gate model was first introduced by Hodgkin and Huxley to describe voltage and time dependence of ion conductances in the squid axon. Today it is still the standard model of the ion current flow through transmembrane channels. One of its main assumptions is that the probability of openning and closing of an ion gate is described by the first-order reaction:

$$\frac{dn}{dt} = \alpha_n(V)(1 - n) - \beta_n(V)n \tag{1}$$

where $\alpha_n(V)$ and $\beta_n(V)$ are voltage-dependent transition rates.

Exercises

1. **Damped harmonic oscillator**

   This exercise is meant to familiarise yourself with numerical solutions to systems of differential equation. As an example we will take a classical damped harmonic oscillator, which is governed by a second-order ordinary differential equation:

$$\ddot{x}(t) + k\dot{x}(t) + \omega^2 x(t) = 0 \tag{2}$$

   with initial conditions $x(t = 0) = 1$, $\dot{x}(t = 0) = 0$.

   (a) Rewrite equation (**??**) as a system of first-order differential equations (*Hint*: use substitution $y = \dot{x}(t)$).

   (b) Write a function which calculates the right-hand side of the two coupled ODEs and returns it as a two-element vector:

   ```
   def harmonic(x_vec, omega=1, k=0.15):
   x, y = x_vec
   ...
   return dxdt, dydt
   ```

(c) Modify your Euler so that it accepts a vectorial function together with a vector of initial conditions and solves the corresponding system of differential equation. Let $\omega = 1$ and $k = 0.15$. Plot the solution.

(d) Use ODE solver included in Python to find the solution:

```
from scipy import integrate
x_0 = [1, 0]; t = arange(0, 5, 0.01)
x = integrate.odeint(harmonic, x_0, t)
```

2. **Potassium channel**

The potassium current in the Hodgkin-Huxley model is given by:

$$I_K = \bar{g}_K n^4 (E_K - V) \tag{3}$$

where $E_K = -77\,\text{mV}$ is the reversal potential and $\bar{g}_K = 36\,\text{mS/cm}^2$ is the maximum conductance. Rates $\alpha_n(V)$ and $\beta_n(V)$ are given by:

$$\alpha_n(V) = 0.01 \frac{V + 55}{1 - \exp(-0.1(V + 55))}, \qquad \beta_n(V) = 0.125 \exp(-0.0125(V+65)). \tag{4}$$

(a) Write a Python function defining $\alpha_n(V)$ and $\beta_n(V)$.

(b) Plot steady-state activation $n_\infty(V) = \alpha_n(V)/(\alpha_n(V) + \beta_n(V))$ and activation time constant $\tau_n(V) = 1/(\alpha_n(V) + \beta_n(V))$ in a voltage range of $-150\,\text{mV} \leq V \leq 150\,\text{mV}$.

(c) Define a system of differential equations describing a potential of membrane with potassium channels. Solve the system for various initial conditions. (*Hint*: use relation $c_m dV/dt = I_K$ with $c_m = 1\,\mu\text{F/cm}^2$ and Equation (??)).

(d) Explain the obtained results refering to the plots of activation variables n_∞ and τ_n.

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